Physics 30 Lesson 1

Momentum and Conservation of Momentum in One Dimension

# Physics principles

Students often ask me if Physics 30 is “harder” than Physics 20. This, of course, depends on the aptitudes, attitudes and work ethic of the individual student. However, there is one major difference between Physics 20 and Physics 30. Physics 20 was dominated by problem solving and the calculation of an answer. Physics 30 has a substantial problem-solving component, but it also requires that students learn and understand the Physics Principles that form the foundation of physics. (These principles are listed on your Physics Data Sheet and are reproduced to the right.) In other words, you will be required to explain how the physics principles are being applied to a particular problem – you will demonstrate that you know the theory behind the problem solving.

**0** Uniform motion 

**1** Accelerated motion 

**2** Uniform circular motion 

**3** Work-energy theorem

**4** Conservation of momentum

**5** Conservation of energy

**6** Conservation of mass-energy

**7** Conservation of charge

**8** Conservation of nucleons

**9** Wave-particle duality

Some of these principles (0, 1, 2, 3 and 5) were taught in Physics 20 and we will see them again in different contexts in Physics 30. The remaining principles are what Physics 30 is all about. This lesson will introduce principle 4, the conservation of momentum.

# Momentum

A very useful physical concept is **momentum**. The momentum () of an object is defined as the product of its mass and velocity. Recall that velocity is a vector quantity that involves both a speed and a direction. Thus, **momentum is a vector quantity**.

 m mass

 velocity

 momentum

There is no chosen unit for momentum like there is for force (N) or energy (J). The unit for momentum is a combination of the mass unit and velocity unit. Some examples are kg·m/s, kg·km/h, g·m/s, etc. (You may refer to Pearson pages 446 to 449 for a different discussion about momentum.)

What is the momentum of a 1500 kg car travelling west at 5.0 m/s?



A micro-meteorite with a mass of 5.0 g has the same momentum as the car in *Example 1*. What is the velocity of the meteorite?



A major misconception that people have is that **inertia** and **momentum** mean the same thing. In the examples above, both objects have the same momentum (mass x velocity) but the inertia (i.e. mass) of the car is substantially larger than the inertia of the meteorite. In other words, inertia refers to the mass of an object while momentum refers to the mass and motion of the object.

# Systems

Before we can understand momentum more fully, we must also be aware of different kinds of **systems**. There are several types of systems:

* **Closed system:** no mass enters or leaves the system
* **Isolated system:** no external forces act on the system and no energy leaves the system.
* **Open system:** mass may enter or leave the system and external forces may influence the system.

To illustrate the difference between these kinds of systems, consider a real-life collision between two cars:

* First, the collision of the cars is **not** an **isolated** system – i.e. they are not isolated from the Earth. Frictional forces between the Earth and the cars will cause the cars to slow down. If the cars collided on a very slippery surface where the frictional forces with the Earth were minimised, this would almost constitute an isolated system.
* Second, if the cars collide and all of the parts of the cars stay attached to the cars we have a **closed** system. However, if parts of the cars fly off we have an **open** system.

In our investigation of the **conservation of momentum** below we will be assuming **closed** and **isolated** systems.

# Conservation of Momentum

The real importance of the concept of momentum is that in any isolated closed system, the **total** momentum of the system is conserved (i.e. remains constant). For example, consider two objects (m1 and m2) that collide as shown below.

Before collision

During collision

After collision



Although the momentum of each individual object changes during the collision, the sum of the momenta before the collision () and after the collision () are the same. The general statement for the **law of conservation of momentum** is:

The total momentum of an isolated system of objects remains constant.

In other words, the sum of the momenta before a collision or explosion (Σ) equals the sum of the momenta after a collision or explosion (Σ).

Note that the prime symbol is used to denote “after” the collision or explosion.



In any collision or explosion, the total momentum is always conserved. This principle proves to be very useful in predicting what will happen when objects collide or explode.

Actually, the principle of the Conservation of Momentum is a direct consequence of Newton’s Third Law of Motion that we learned about in Physics 20. Recall that when any object exerts a force on another object, the second object will exert an equal and opposite force on the first object. Consider the collision between the two masses illustrated above. When they collide, mass 1 exerts a force on mass 2 () resulting in an acceleration (change in velocity) of mass 2. According to Newton’s 3rd Law, if object 1 exerts a force on object 2, object 2 will exert an equal and opposite reaction force on object 1 () resulting in a change in velocity of mass 2. Therefore



If we apply Newton’s Second Law of Motion 



the time of contact is the same for both objects

∴ t cancels out

Notice that when the change in momentum of 1 and 2 are added together, the result is zero.



This result indicates that while the momentum of each object changes, **the total change in momentum of the system (i.e. both objects) is zero**. This is also a statement of the principle of **conservation of momentum**.

# Elastic and inelastic collisions

There are two basic types of collisions – elastic and inelastic – and we use the law of conservation of momentum to predict and calculate the results.

**Elastic** collisions are those where **both** momentum and kinetic energy are conserved. Purely elastic collisions are very hard to produce in the ordinary world because there is always some kinetic energy converted into heat, sound, deformation or some other form of energy. As we shall see in future lessons on the nature of the atom, collisions between subatomic particles are elastic collisions.

**Inelastic** collisions are those where momentum is conserved, but kinetic energy is not conserved. A **completely inelastic** collision is when the objects collide and stick together.

A 2.00 kg object travelling east at 4.00 m/s collides with a 3.00 kg object travelling west at 1.50 m/s. If the 3.00 kg object ends up travelling east at 1.25 m/s, what is the final velocity of the 2.00 kg object?

Before After

= – 1.50 m/s

= + 4.00 m/s



= +1.25 m/s



m1 = 2.00 kg m2 = 3.00 kg

m1 = 2.00 kg m2 = 3.00 kg

When we apply the conservation of momentum principle we want the equation to reflect the context of the question. In this question there are two objects before the collision and two objects after the collision. Therefore there are two  terms on the before side and two  terms on the after side.

Rearrange the equation, plug in the known values, and calculate the unknown.



A 30 kg object travelling at 45 m/s west collides with a 40 kg object at rest. If the objects stick together on contact, what is the resulting velocity of the combined masses?

Before After

= -45 m/s



m1+2 = 40 kg + 30 kg

m2 = 40 kg m1 = 30 kg



In this question there are two objects before the collision and one object after the collision. Therefore there are two  terms on the before side and one  term on the after side.

This is an example of a **completely inelastic** collision.

A 400 kg object travelling east at 50 m/s collides with a moving 50 kg object. The masses stick together after they collide and move at 27.8 m/s to the east. What was the initial velocity of the 50 kg object?

Before After

= + 50 m/s = ?

= + 27.8 m/s

m1+2 = 400 kg + 50 kg

m1 = 400 kg m2 = 50 kg



A 2.00 kg object travelling east at 4.00 m/s collides with a 3.00 kg object travelling west at 1.50 m/s. If the 3.00 kg object ends up travelling east at 2.10 m/s.

a. What is the final velocity of the 2.00 kg object?

b. Was the collision elastic?

c. How can momentum always be conserved while kinetic energy is not always conserved?

a. Use the conservation of momentum to calculate the final velocity of the 2.00 kg object.

Before After

m1 = 2.00 kg m2 = 3.00 kg

= + 4.00 m/s





= +2.10 m/s

m1 = 2.00 kg m2 = 3.00 kg

= – 1.50 m/s



b. To determine if the collision was elastic, calculate the kinetic energy before the collision and after the collision and compare the values.



Since the final kinetic energy is less than the initial kinetic energy, the collision is **inelastic**.

c. Momentum in an isolated system is **always** conserved. This law is a direct result of Newton’s third law of motion where the change in momentum of one object is equal and opposite the change in momentum of the other object. The combined change in momentum is zero.

Kinetic energy, on the other hand, can easily be transformed into heat, sound, and the deformation of objects. The Conservation of Energy (i.e. − the total amount of energy in an isolated system does not change, but energy can be transformed into other types) does not depend on Newton’s Laws.

# Explosions

The primary difference between collisions and explosions is that prior to an explosion there is one object, and after the explosion there are several objects.

A 5.0 kg rifle fires a 0.020 kg bullet to the east. If the muzzle speed of the bullet is 400 m/s, what is the recoil velocity of the rifle?

Note that initially the bullet and rifle are at rest. Therefore Σ = 0



A 4.00 kg bomb is rolling across a floor at 3.00 m/s to the east. After the bomb explodes a 1.50 kg piece has a velocity of 15.0 m/s east. What is the velocity of the other piece?



In the diagram below, ball A (mass = 5.0 kg) is traveling to the left at 12 m/s and collides with a stationary ball B (mass = 4.0 kg). After the collision ball A is moving at 0.50 m/s to the left. How high up the incline will ball B rise before coming to rest?

B

A

20o

d

The first part of the problem involves a collision between ball A and ball B. Thus we use the conservation of momentum to find the final speed of ball B. Once we know the speed of ball B, we can use the conservation of energy to determine how high it rises on the incline.

Part A – Find vB’



Part B – Find the height up the ramp

The kinetic energy of ball B after the collision is converted into potential energy (h).



# Practice Problems

1. A 0.25 kg ball travelling east at 4.5 m/s collides with a 0.30 kg steel ball travelling west at 5.0 m/s. After the collision the 0.30 kg ball is travelling east with a speed of 0.40 m/s. What is the final velocity of the 0.25 kg ball? (1.98 m/s west)

2. A freight train is being assembled in a switching yard. Boxcar #1 has a mass of 6.4 x 104 kg and moves with a velocity of +0.80 m/s. Boxcar #2, with a mass of 9.2 x 104 kg and a velocity of +1.2 m/s, overtakes Boxcar #1 and couples with it. Neglecting friction, find the common velocity of the two cars after they have coupled. (+1.0 m/s)

3. Starting from rest, two skaters “push off” against each other on smooth, level ice. One is a woman (m = 54 kg) and the other is a man (m = 88 kg). After pushing off, the woman moves away with a velocity of +2.5 m/s. Find the recoil velocity of the man. (–1.5 m/s)

# Hand-in Assignment

1. Explain, in your own words, the difference between momentum and inertia.

2. What is the momentum of a 6.0 kg bowling ball with a velocity of 2.2 m/s [S]? (13.2 kg·m/s [S])

3. The momentum of a 75 g bullet is 9.00 kg•m/s [N]. What is the velocity of the bullet? (120 m/s [N])

4. A hockey puck has a momentum of 3.8 kg•m/s [E]. If its speed is 24 m/s what is the mass of the puck? (1.6 x 10-1 kg)

5. A jet flies west at 190 m/s.

(a) What is the momentum of the jet if its total mass is 2250 kg?

(4.28 x105 kg·m/s [W])

(b) What would be the momentum of the jet if the mass was 4 times its original value and the speed increased to 6 times its original value?

(1.03 x107 kg·m/s [W])

6. A 30.0 kg object moving to the right at a velocity of 2.00 m/s collides with a 20.0 kg object moving to the left with a speed of 6.00 m/s. If the 20.0 kg object rebounds to the right with a speed of 0.75 m/s, what is the final velocity of the 30.0 kg object? (2.50 m/s [left])

7. A 225 g ball with a velocity of 40.0 cm/s [right] collides with a 125 g ball moving with a velocity of 15.0 cm/s [right]. After the collision the velocity of the 125 g ball was 35.0 cm/s [right]. What was the velocity of the 225 g ball after the collision? Was the collision elastic or inelastic? (28.9 cm/s [right], inelastic)

8. A 925 kg car moving with a velocity of +20.0 m/s collides with a stationary truck of unknown mass. The vehicles lock together and move off with a velocity of +6.75 m/s. What was the mass of the truck? (1.82 x 103 kg)

9. In a football game, a receiver catches a ball while he is standing still. Before he can move, a tackler, running at 4.75 m/s, grabs him. The tackler holds onto the receiver and the two move off with a speed of 2.50 m/s. If the mass of the tackler is 125 kg, what is the mass of the receiver? (113 kg)

10. An arrow travelling at 45 m/s strikes and embeds itself in a 450 g apple which is initially at rest. They move off horizontally at 12 m/s after impact. What is the mass of the arrow? (164 g)

11. A 1.0 x 105 N truck moving at a velocity of 17 m/s north collides head on with a 1.0 x 104 N car moving at 29 m/s south. If they stick together, what is the final velocity of the car? (12.8 m/s north)

12. An astronaut is motionless in outer space. Upon command, the propulsion unit strapped to her back ejects gas with a velocity of +16 m/s causing the astronaut to recoil with a velocity of –0.55 m/s. After the gas is ejected, the astronaut has a mass of 160 kg. What is the mass of the ejected gas? (5.5 kg)

13. A two-stage rocket moves in space at a constant velocity of +4900 m/s. The two stages are then separated by a small explosive charge placed between them. Immediately after the explosion the velocity of the 1200 kg upper stage is +6000 m/s. What is the velocity of the 2400 kg lower stage after the explosion? (+4350 m/s)

14. In the James Bond movie *Diamonds are Forever* the lead female character fires a machine gun while standing on the edge of an off-shore drilling rig. As she fires the gun she is driven back over the edge into the sea. The mass of a bullet is 10 g and its velocity is 750 m/s. If her mass (including the gun) is 55 kg, what recoil velocity does she acquire in response to a single shot from a stationary position? (–0.14 m/s)

15. An atom of uranium disintegrates into two particles. One particle has a mass 60 times as great as the other. If the larger particle moves to the left with a speed of 2.3 x 104 m/s, with what velocity does the lighter particle move? (+1.4 x 106 m/s)

16. A ballistic pendulum is a laboratory device which might be used to calculate the velocity of a bullet. A 5.0 g bullet is caught by a 500 g suspended mass. After impact, the two masses move together until they stop at a point 65 mm above the point of impact. What was the velocity of the bullet prior to impact? How much of the original kinetic energy of the bullet energy was converted into heat?

65 mm

V = ?

(114 m/s, -32.2 J)

\*17. A 70 kg boy sits in a 30 kg canoe at rest on the water. He holds two cannon balls, each of mass 10 kg. He picks them up and throws both together to the stern of the canoe. The two balls leave his hands with a velocity of 5.0 m/s *relative to the canoe*.

A 50 kg girl sits in a 50 kg canoe also at rest on the water. She also holds two 10 kg cannon balls. However she throws them over the stern of her canoe one at a time, each ball leaving her hands with a velocity of 5.0 m/s *relative to the canoe*.

Assuming negligible friction between the water and the canoes (a poor assumption), calculate the final velocity for each canoe. (Hint: Describe the velocities relative to the water, not the canoes.) (0.83 m/s, 0.87 m/s)

\*18. Two men, each with a mass of 100 kg, stand on a cart of mass 300 kg. The cart can roll with negligible friction along a north-south track and everything is initially at rest. One man runs toward the north and jumps off the cart with a speed of 5.00 m/s *relative to the cart*. After he has jumped, the second man runs south and jumps off the cart again with a speed of 5.0 m/s *relative to the cart*. Calculate the speed and direction of the cart after both men have jumped off. (0.25 m/s North)

\*19. An argon atom with a mass of 6.680 x 10-26 kg travels at 17.00 m/s [right] and elastically strikes an oxygen atom with a mass of 2.672 x 10-26 kg dead centre travelling at 20.00 m/s [left]. What are the final velocities of each atom?

(4.143 m/s [left], 32.86 m/s [right])

Momentum in One Dimension Activity

## Getting started

This activity involves the use of an applet. Google **phet** from the University of Colorado. Find the **Collisions** applet and download/run it.

* Use the Introduction applet. We will use the Advanced one in the next lesson.
* Play with the applet for a while. Learn how to manually set the mass and velocity of each puck and to set the “elasticity” (e) value. When e = 1 both momentum and kinetic energy are conserved. When e = 0 the pucks stick together. When e has a value between 0 and 1, momentum is conserved but kinetic energy is not conserved.

## Conservation of Momentum

Any object that is moving has velocity, and also momentum. But what happens if two or more objects collide? What happens to the velocity or the momentum of each object? Let's explore this question with the applet.

Perform three different collisions

* For each collision, make sure that each puck has a different mass.
* One collision should involve the red puck coming to a stop or continuing to move in its original direction.
* One collision should involve the red puck rebounding back in the opposite direction.
* One collision should involve the pucks sticking together.

and complete the following tables:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Collision 1** | | | | | | | |
| Object | mass (kg) | (m/s) | (m/s) | (m/s) | (kg·m/s) | (kg·m/s) | (kg·m/s) |
| **Red** |  |  |  |  |  |  |  |
| **Green** |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Collision 2** | | | | | | | |
| Object | mass (kg) | (m/s) | (m/s) | (m/s) | (kg·m/s) | (kg·m/s) | (kg·m/s) |
| **Red** |  |  |  |  |  |  |  |
| **Green** |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Collision 3** | | | | | | | |
| Object | mass (kg) | (m/s) | (m/s) | (m/s) | (kg·m/s) | (kg·m/s) | (kg·m/s) |
| **Red** |  |  |  |  |  |  |  |
| **Green** |  |  |  |  |  |  |  |

Now, look at the information you recorded and calculated in the previous question. What relationships exist between what happens to the red mass and the green mass?

1. Is there a relationship between the change in velocity of the red mass and the green mass? If yes, describe the relationship.
2. Is there a relationship between the change in momentum of the red mass and the green mass? If yes, describe the relationship.
3. You should see an interesting connection between the change in momentum of each mass. If the change in momentum of one object is exactly equal, but opposite to the change in momentum of another object, what does that indicate about the total momentum of the system?

If the total momentum of a system before a collision is equal to the total momentum of a system after a collision, then we say that **momentum is conserved**.   For the three collisions verify that the total initial momentum is equal to the total final momentum:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Note: | |  | | |
|  | |  | | |
| Collision # | | total initial momentum  (kg·m/s) | | total final momentum  (kg·m/s) | Is momentum conserved? | |
| **1** | |  | |  |  | |
| **2** | |  | |  |  | |
| **3** | |  | |  |  | |